CALCULUS.

Solutions of Problem 172 have been received from J. E. Sanders, Hackney, Ohio, and J. B. Gregg, M. Sc., C. E., Senecaville, Ohio.

173. Proposed by J. E. SANDERS, Hackney, Ohio.

Find the greatest ellipse that can be inscribed in a quadrant of a given circle.

Solution by J. B. GREGG, M. Sc., C. E., Senecaville, Ohio, and G. W. GREENWOOD, B. A. (Oxon), Professor of Mathematics and Astronomy, McKendree College, Lebanon, Ill.

By symmetry, the radius bisecting the quadrant will coincide with an axis of the ellipse. Denote the length of this semi-axis, and that of the transverse semi-axis, by a, b. Then

πab=area of ellipse$=A$ (say).

$a+\sqrt{(a^2+b^2)}$=radius of given circle$=r$ (say).

$\therefore r^2=2ar+b^2$.

Since A is a maximum, $bda+adb=0$. Also $rda+bdb=0$.

$\therefore ra=b^2$, $r=3a$, $A=\frac{1}{9}\pi r^2\sqrt{3}$.

Also solved by G. B. M. Zerr, A. M., Ph. D., Parsons, W. Va., and A. H. Holmes, Brunswick, Me.

MECHANICS.

163. Proposed by W. J. GREENSTREET, A. M., Editor of The Mathematical Gazette, Stroud, England.

A particle A, mass m, rests on a smooth horizontal plane and is attached by two inelastic strings to masses m_1, m_2 at points B and C such that BAC is a right angle. If a blow is given A at an angle θ to AB, find the initial direction of motion of m, and equations for initial motion of the particles m_1 and m_2.

Solution by J. B. GREGG, M. Sc., C. E., Senecaville, Ohio.

Let v, v_1, and v_2 be the respective initial velocities of m, m_1, and m_2, and let ϕ be the angle which the initial motion of m makes with the line of direction of the blow. Construct $AD=v$; then DE perpendicular to BA produced$=v_2$, $AE=v_1$, $\angle DAE=\theta+\phi$.

$v_1=v\cos(\theta+\phi)$, $v_2=v\sin(\theta+\phi)$, $mv\sin\phi+m_1v_1\sin\theta=m_2v_2\cos\theta$.

Solving for ϕ, $\tan\phi=\dfrac{(m_2-m_1)\sin\theta\cos\theta}{m-m_1\sin^2\theta-m_2\cos^2\theta}$.

DIOPHANTINE ANALYSIS.

116. Proposed by HARRY S. VANDIVER, Bala, Pa.

If n is an odd positive integer, and $1, n, n', n''$, ... denote all its distinct divisors, then $2^n>2(n+1)(n'+1)(n''+1)$.....

II. Solution by the PROPOSER.

By Euler's generalization of Fermat's Theorem (Dirichlet-Dedikind *Zah-*

lentheorie, 4th edition, p. 38), we have for n odd,

$$2^{\phi(1)}-1\equiv 0(\mathrm{mod}1),\ 2^{\phi(n)}-1\equiv 0(\mathrm{mod}n),\ 2^{\phi(n')}-1\equiv 0(\mathrm{mod}n'),\ldots\ldots\ldots$$

where $\phi(n)$ is the number of integers less than n and prime to it. We have, then, since $\phi(1)=1$,

$$2^{\phi(1)}-1=1,\ 2^{\phi(n)}-1\geqq n,\ 2^{\phi(n')}-1\geqq n',\ldots\ldots\ldots$$

Now, unless $n=3$, there is at least one of the latter relations which is not an equality. For, let p be the largest prime factor of n, then $2^{p-1}-1\geqq p$.

Make $p=3$, then $2^{3-1}-1=3$. From this relation it is evident that $2^{p-1}-1>p$ for $p>3$. If $p=3$, then $n=3^k$. But

$$2^{\phi(3^2)}-1>3^2.$$

Hence, unless $k=1$ or $n=3$, there is at least one inequality in the original scheme, and we get by transposition and multiplication,

$$2^{\phi(1)+\phi(n)+\phi(n')\ldots\ldots}>2(n+1)(n'+1)(n''+1)\ldots\ldots\ldots$$

or, since $\phi(1)+\phi(n)+\phi(n')\ldots\ldots=n$ (Dirichlet, l. c., p. 26),

$$2^n>2(n+1)(n'+1)(n''+1)\ldots\ldots\ldots$$

for $n>3$, which is the theorem.

118. Proposed by L. C. WALKER, A.M., Professor of Mathematics, Colorado School of Mines, Golden, Col.

Find the two least *positive** integral numbers such that their sum shall be a square and the sum of their squares a biquadrate.

Professor Walker finds that the two numbers,

$$x=4{,}565{,}468{,}027{,}761,\quad y=1{,}061{,}625{,}293{,}520$$

have the properties that their sum is a square and the sum of their sqnares is a biquadrate. Indeed, it may be verified that $(x+y)^{\frac{1}{2}}=2372159$; $(x^2+y^2)^{\frac{1}{4}}=2165017$. In the letter accompanying the solution Professor Walker asks "are these the least numbers?" We invite other solutions bearing upon this important point of the problem.

119. Proposed by L. E. DICKSON, Ph. D., Assistant Professor of Mathematics, The University of Chicago.

If p be any prime number and n any positive integer, the congruence $x^{p^n}\equiv x(\mathrm{mod}p^n)$ has p and only p solutions $\mathrm{mod}p^n$. Hence the congruence defines the Galois field of order p^n if and only if $n=1$.

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*As originally printed the word "positive" was omitted.